Single Pure - Graphical Transformations Worksheet

For a curve, $y = f(x)$, we have the following transformations:

$y = af(x)$	:	A stretch (with x-axis invariant) parallel to the y-axis of scale factor a.
$y = f(x) + a$	:	A translation of a units in the positive y direction (translation vector $\begin{pmatrix}0\\a\end{pmatrix}$).
$y = f(ax)$	:	A stretch (with y-axis invariant) parallel to the x-axis of scale factor $\frac{1}{a}$.
$y = f(x + a)$	:	A translation of a units in the negative x direction (translation vector $\begin{pmatrix}-a\\0\end{pmatrix}$).
$y = -f(x)$	:	A reflection in the x-axis.
$y = f(-x)$	:	A reflection in the y-axis.

Whenever you see a quadratic it is usually advantageous to complete the square.

Questions

1. Describe the transformation(s) that map the first curve onto the second:

   (a) $y = 3x + 1$ onto:

   i. $y = -3x - 1$. — Reflection in x-axis

   ii. $y = 6x + 2$. — Stretch, sf 2 parallel to y-axis

   iii. $y = 6x + 1$. — Stretch, sf $\frac{1}{2}$ parallel to x-axis

   iv. $y = -3x + 1$. — Reflection in y-axis

   (b) $y = x^2 + 1$ onto:

   i. $y = x^2 - 1$. — Translation by $\begin{pmatrix}0\\-2\end{pmatrix}$

   ii. $y = x^2 + 2x + 3$. — Translation by $\begin{pmatrix}-1\\1\end{pmatrix}$

   iii. $y = 2x^2 + 2$. — Stretch, sf 2 parallel to y-axis

   iv. $y = 4x^2 + 1$. — Stretch, sf $\frac{1}{2}$ parallel to x-axis

   v. $y = -x^2 - 1$. — Reflection in x-axis

   (c) $y = \frac{2}{x-1}$ onto:

   i. $y = \frac{2}{x}$. — Translation by $\begin{pmatrix}-1\\0\end{pmatrix}$

   ii. $y = \frac{1}{x-1}$. — Stretch, sf $\frac{1}{2}$ parallel to y-axis

   iii. $y = \frac{2}{x-4} + 2$. — Translation by $\begin{pmatrix}3\\2\end{pmatrix}$

   iv. $y = \frac{2}{3x-1}$. — Stretch, sf $\frac{1}{3}$ parallel to x-axis

   v. $y = -\frac{2}{x+1}$. — Reflection in y-axis

   (d) $y = \sqrt{x}$ onto:

   i. $y = \sqrt{x+3} - 2$. — Translation by $\begin{pmatrix}-3\\-2\end{pmatrix}$

   ii. $y = \sqrt{-x}$. — Reflection in y-axis

   iii. $y = -\sqrt{x}$. — Reflection in x-axis

   iv. $y = \sqrt{\frac{x}{4}}$. — Stretch, sf 4 parallel to x-axis

   v. $y = \pi\sqrt{x}$. — Stretch, sf π parallel to y-axis

2. Describe *two* possible single transformations that map:

   (a) $y = x^2$ onto $y = 9x^2$.

   (b) $y = 2^x$ onto $y = 2^{x+1}$.

(c) $y = \sqrt{x}$ onto $y = \sqrt{3x}$.

3. Find the equation of new curves after the following transformations:

(a) $y = x^2 + 3x - 1$ after:

i. the translation $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$. $y = x^2 + 3x + 4$

ii. the translation $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$. $y = x^2 - x - 3$

iii. the translation $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$. $y = x^2 + 9x + 13$

iv. a stretch parallel to the y-axis, scale factor 5. $y = 5x^2 + 15x - 5$

v. a stretch parallel to the x-axis, scale factor 3. $y = \frac{x^2}{9} + x - 1$

(b) $y = -x^2 + x + 1$ after:

i. a reflection in the x-axis. $y = x^2 - x - 1$

ii. a reflection in the y-axis. $y = -x^2 - x + 1$

iii. a stretch parallel to the y-axis, scale factor $\frac{1}{3}$. $y = -\frac{x^2}{3} + \frac{x}{3} + \frac{1}{3}$

(c) $y = x + \frac{1}{x+3}$ after:

i. a stretch parallel to the y-axis scale factor 4. $y = 4x + \frac{4}{x+3}$

ii. a reflection in the x-axis. $y = -x - \frac{1}{x+3}$

iii. a translation $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$. $y = x + \frac{1}{x} - 4$

iv. a stretch parallel to the x-axis scale factor 2. $y = \frac{x}{2} + \frac{2}{x+6}$

(d) $y = x^3 + x^2 + x$ after:

i. a stretch parallel to the x-axis scale factor 2. $y = \frac{x^3}{8} + \frac{x^2}{4} + \frac{x}{2}$

ii. a reflection in the x-axis. $y = -x^3 - x^2 - x$

iii. the translation $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$. $y = (x+3)^3 + (x+3)^2 + (x+3) + 4$

iv. a reflection in the y-axis. $y = -x^3 + x^2 - x$

v. the translation $\begin{pmatrix} a \\ 0 \end{pmatrix}$. $y = (x-a)^3 + (x-a)^2 + (x-a)$

(e) $y = \sqrt{2x+3}$ after:

i. the translation $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$. $y = \sqrt{2x+5} + 2$

ii. a stretch parallel to the y-axis scale factor 3. $y = 3\sqrt{2x+3}$

iii. a reflection in the x-axis. $y = -\sqrt{2x+3}$

iv. a reflection in the y-axis. $y = \sqrt{3-2x}$

v. a stretch parallel to the x-axis scale factor 2. $y = \sqrt{x+3}$

4. (a) Sketch the graph $y = \begin{cases} x & 0 \leqslant x < 1 \\ 2 - x & 1 \leqslant x \leqslant 2 \\ 0 & \text{otherwise.} \end{cases}$

(b) Sketch $y = f(x+1) - 1$.

(c) Sketch $y = f(\frac{x}{3})$.

(d) Sketch $y = -2f(x - \pi)$.

(e) The equation $f(x) = k$ has an infinite number of solutions. Find the possible value(s) of k.

5. (a) Sketch the graph $y = \begin{cases} x(x+2) & -2 \leqslant x < 0 \\ x(2-x) & 0 \leqslant x \leqslant 2 \\ 0 & \text{otherwise.} \end{cases}$

(b) Sketch $y = f(x - 3) + 1$.

(c) Sketch $y = 2f(x + 1)$.

(d) Sketch $y = -f(\frac{x}{2})$.

(e) Sketch $y = f(-2x)$.

(f) The equation $f(x) = k$ has exactly two distinct solutions. Find the possible value(s) of k.